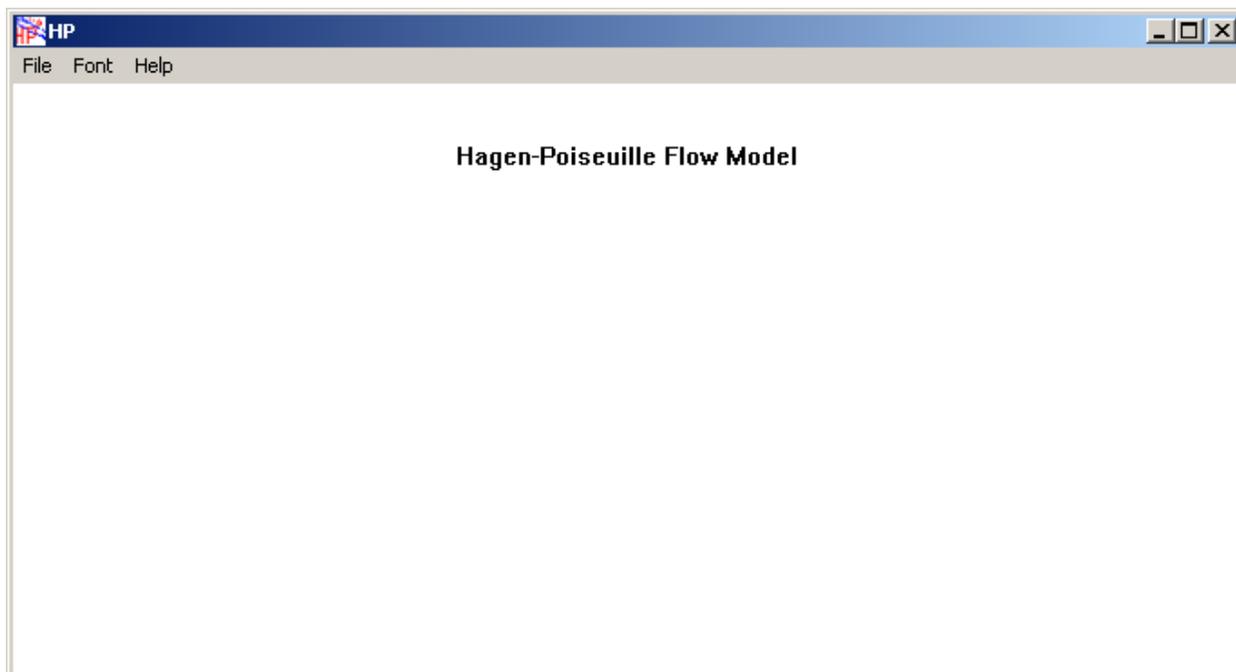User Notes for the "HP" Program

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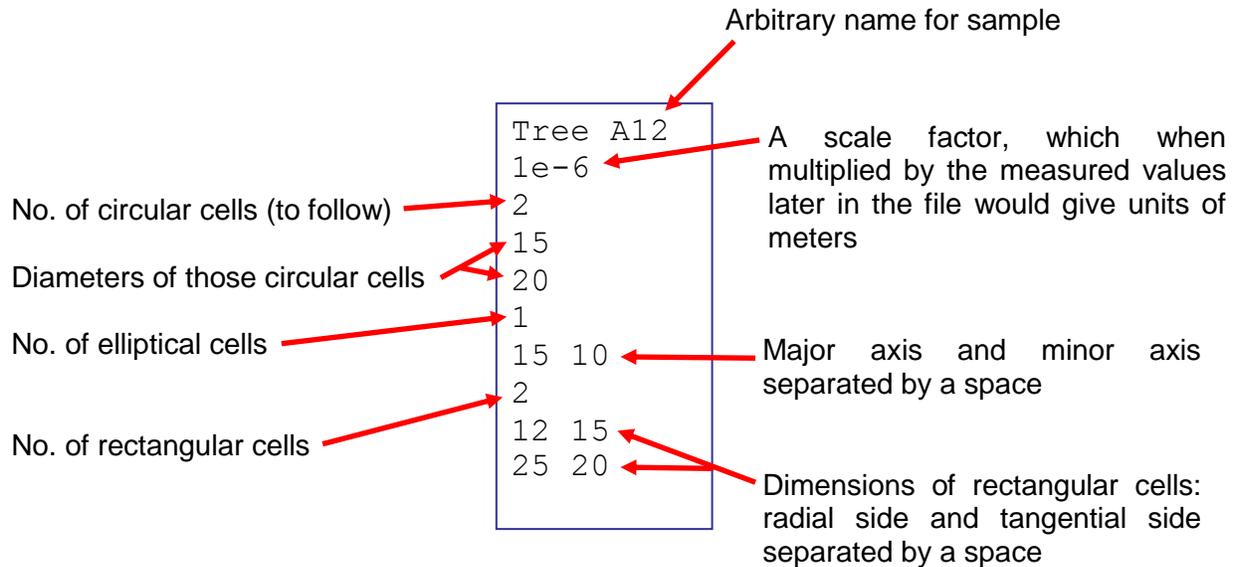University of Nevada, Las Vegas

Introduction

Studies of water flow through the xylem of plants may involve measurements of the diameters of the tracheids or vessels as conduits. A useful statistic might be the average diameter of these cells and a predicted hydraulic conductance. But the cell cross-sections are often not circular. Fluid mechanics provides us with a number of formulae describing flow through conduits with non-circular cross-sections (see Appendix) and so one approach would be to convert all cells to circular conduits of equivalent conducting ability. This program reads through a data file containing a list of conduit measurements, performs the necessary circularization, and calculates a predicted conductance based on the Hagen-Poiseuille equation.

Running the program

Upon startup, the program creates a window like that shown here. The first step is likely to be selecting a data file for input. The expected format for this file may be seen by selecting "Help…File format…" from the program's menu system.

Here is a sample data file:

Arbitrary name for sample

```
Tree A12
1e-6
2
15
20
1
15 10
2
12 15
25 20
```

A scale factor, which when multiplied by the measured values later in the file would give units of meters

No. of circular cells (to follow)

Diameters of those circular cells

No. of elliptical cells

Major axis and minor axis separated by a space

No. of rectangular cells

Dimensions of rectangular cells: radial side and tangential side separated by a space

The length of the file will vary of course depending on the number of cells measured - the number of cells for each shape is entered before that group of cells. The program does a little bit of input file error checking, such that if the numbers of row of cell data is less than the numbers of cells specified, an error message will be presented.

The original idea with rectangular cells (unlike for the elliptical cells) was to maintain some info about the orientation of those cells. So their dimensions can be entered as the cell width along the radial face and tangential face because most of the rectangular cells that I have seen have their walls oriented this way. A related program called "diameters" keeps track of both radial & tangential dimensions as well as long side & short side dimensions to give a sense (on average) if the cells have their long or short sides in the radial or tangential orientation. Such considerations of orientation do not affect the circularization calculations, and so for this program you can always enter data for rectangular conduits in any order.

Data files are selected with the "File…Open…" menu item. The program uses a standard MS Windows file dialog, where you can select files for input. The program is expecting data files to have a "dat" extension to the filename.

After data file has been read, the program window will display all the statistics and predicted conductance.
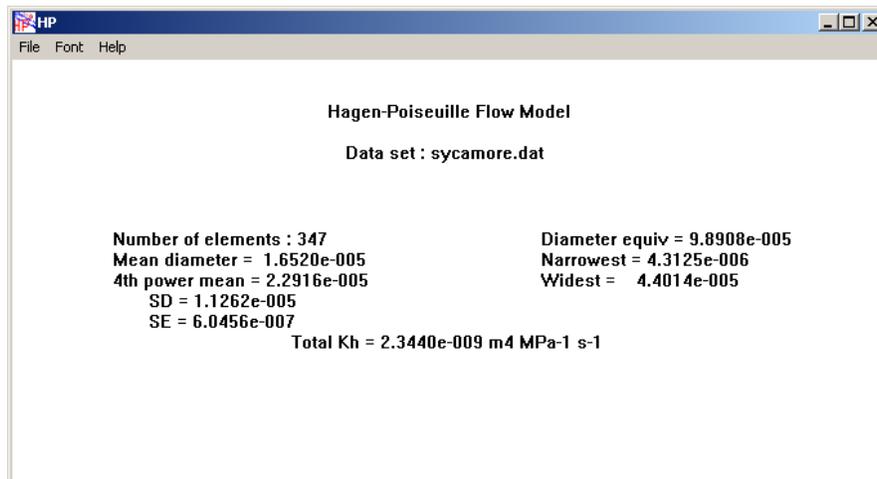
Program output

Here's a typical program window - in this case after reading a sample data file called "sycamore.dat".

Mean 4th diameter uses a formula like:

$$d^* = \sqrt[4]{\frac{\sum_{i=1}^{N} d_i^4}{N}}$$

that some users may feel is a sensible approach because for simple, ideal pipes, conducting ability is related to the fourth power of diameter. In this manner, the mean is "weighted" more heavily towards wider cells because they presumably contribute more to flow than narrow cells.

HP

File   Font   Help

Hagen-Poiseuille Flow Model

Data set : sycamore.dat

Number of elements : 347   Diameter equiv = 9.8908e-005
Mean diameter = 1.6520e-005   Narrowest = 4.3125e-006
4th power mean = 2.2916e-005   Widest =   4.4014e-005
     SD = 1.1262e-005
     SE = 6.0456e-007
              Total Kh = 2.3440e-009 m4 MPa-1 s-1

The value referred to as "Total Kh" is the total hydraulic conductance per unit length for all cells (in parallel) as predicted by the Hagen-Poiseuille equation:

$$K_h = \sum_{i=1}^{N} \frac{\pi d_i^4}{128\eta}$$

where d is the circularized cell diameter, η is the fluid viscosity (assumed to be 1.0021E-9 MPa s at 20 ⁰C), and N is the total number of cells.

The value referred to as "Diameter equiv" is the diameter of a single circular conduit that would provide the equivalent conductance of all of cells provided in the data file (assuming of course that the conduits behave as ideal pipes).

All dimensions are given in meter units assuming the correct scale factor was provided in the data file.

The "Font" menu item allows you to select the font type and size used for display in the program window. The "Save settings" menu item will save the current program window size and location on your display along with the current font choice to a file called "HP.ini" located in the same place as the program. When the program starts, it looks for

a file with that name and uses those settings. If no such file exists, default values are used. In this manner, you can setup the program window size and location as you would prefer and have those settings remembered.

Appendix

Calculation of equivalent circular conduits from elliptical and rectangle conduits

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Tracheids of Douglas-fir observed in cross-section were rarely circular, with elliptical and rectangular cells occurring in roughly equal numbers. A common basis for comparing tracheid sizes was needed and because water transport is one of the primary functions of these tracheids, conducting ability was chosen as a basis. The hydraulic conductance (K) of simple conduits is nonlinearly related to the size of the conduit and is given for these shapes as (see Langlois, 1964 and White, 1991):

$$K^{circular} = \frac{\pi d^4}{128\eta} \tag{1}$$

where d is the conduit diameter and η is the fluid viscosity;

$$K^{elliptical} = \frac{\pi}{4\eta}\frac{(ab)^3}{(a^2+b^2)} \tag{2}$$

where a is the semi-major axis and b is the semi-minor axis;

$$K^{rectangular} = \frac{4ba^3}{3\eta}\left(1 - \frac{192a}{\pi^5 b}\sum_{i=1,3,5}^{\infty}\frac{\tanh(i\pi b/2a)}{i^5}\right) \tag{3}$$

where, here, a is the semi-long side and b is the semi-short side of the rectangle. Equivalent diameters for elliptical conduits can be calculated by equating equations 1 and 2, and for rectangular conduits by equating equations 1 and 3, giving:

$$d = \left(\frac{32(ab)^3}{(a^2+b^2)}\right)^{1/4} \qquad \text{(for ellipses)} \tag{4}$$

$$d = \left[\frac{512ba^3}{3\pi}\left(1 - \frac{192a}{\pi^5 b}\sum_{i=1,3,5}^{\infty}\frac{\tanh(i\pi b/2a)}{i^5}\right)\right]^{1/4} \qquad \text{(for rectangles)} \tag{5}$$

Fortunately, the series summation in Equation (5) converges such that 32 terms (i up to 65) gives a sum accurate to at least 7 significant figures for rectangular conduits with

sides ranging from 1:1 to 10:1. A program was developed (using Microsoft Visual C++ for Windows) to carry out these calculations along with statistical data such as mean cell diameter and standard deviation.

Literature Cited

Langlois, WE. 1964. Slow Viscous Flow. Macmillan, New York.

White, FM. 1991. Viscous Fluid Flow. McGraw-Hill, New York.